Regression Error

Grinnell College

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Review

- Regression models a linear relationship between response variable y and explanatory variable X of the form

$$y = \beta_0 + \beta_1 X + \epsilon$$

- Can expand this to include combinations of explanatory variables (quant. and cat.)

Error Terms

$$y = \beta_0 + X\beta_1 + \epsilon$$

Assumptions:

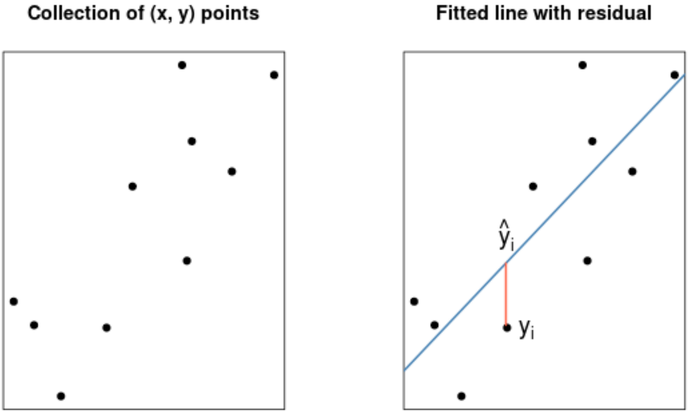
- Linear relationship between X and y
- Error term is normally distributed, $\epsilon \sim N(0, \sigma)$
  - We needed Normal distributions for means when using t-tests
- Error *variance* should be the same for all values of X, i.e., roughly same same error for all observations
  - otherwise something could be going horribly wrong

Graphing the residuals gives us a way to test the assumptions of our model

Residuals

Visually, let's review what residuals look like

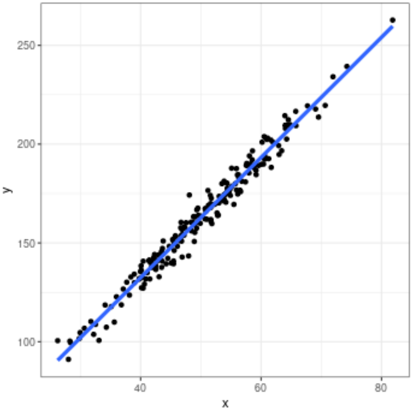
- residuals represent how far off our prediction is



Residuals and assumptions

Three common ways to investigate residuals visually:

1. Plot histogram of residuals (normality)
2. Plot residuals against a predictor (linear trend, changing variance)
3. Normal Quantile Plot – compares quantiles of residuals to quantiles of Normal distribution to see if they match

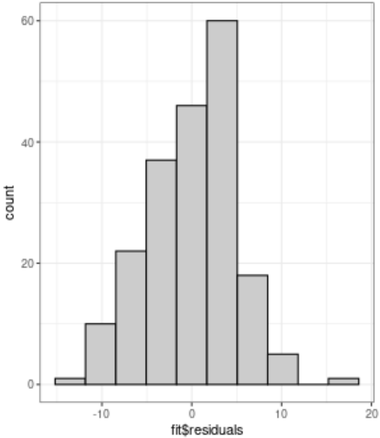


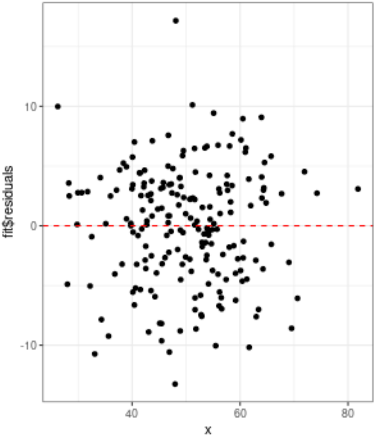
Checking Normality

Histogram of Residuals should be ≈Normal if our model is doing well

Residuals should not have a pattern other than 'blob of points' in a Resid. vs. Expl. Var. scatterplotplot

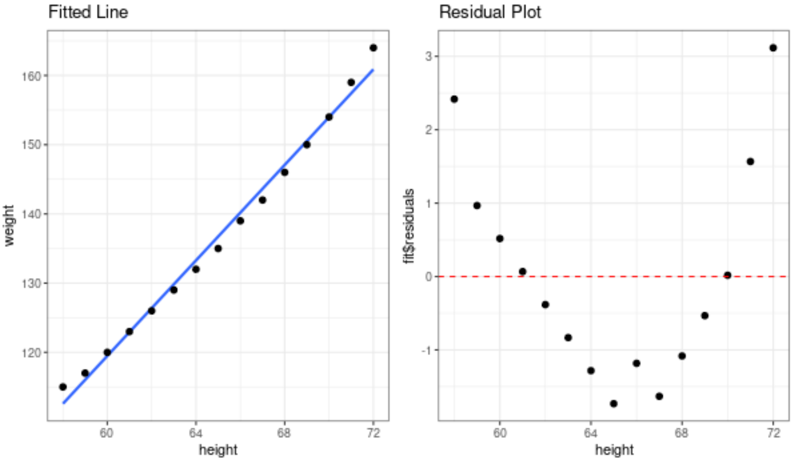
- don't want correlation between residuals and explanatory variables





Tests of linearity

Residual vs. Explanatory plot makes seeing non-linearity easier

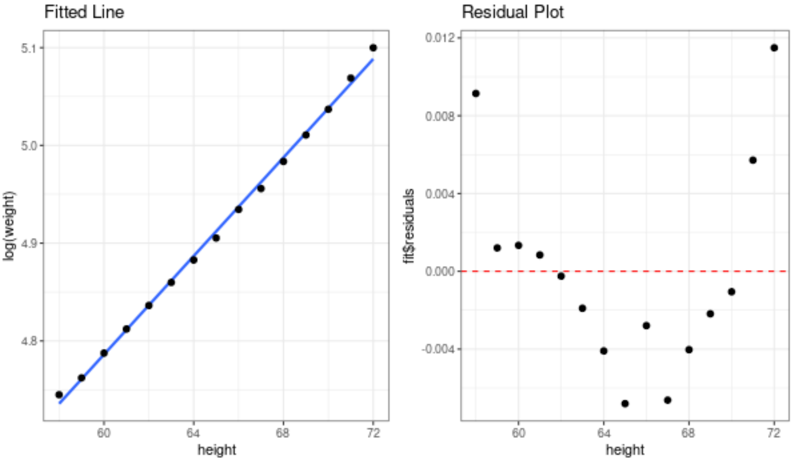


- linear regression could still be useful!
- but we could also look at doing something more complicated if we really cared

Tests of linearity

Sometimes a transformation of a variable can help correct trends $\rightarrow$ log(weight)

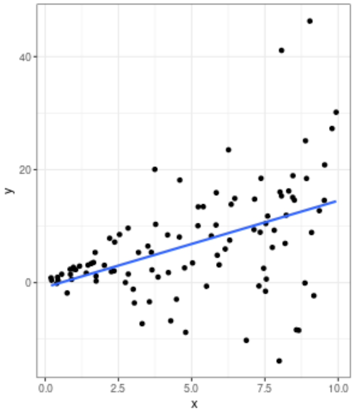
- better, still have a funky Residual vs. Height plot

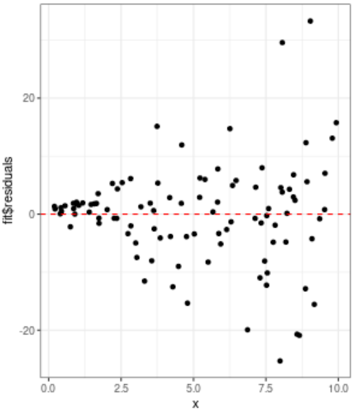


Heteroscedasticity / Homoscedasticity

Hetero- = different, Homo- = same, scedastic = random
We do not want variance of residuals to increase for really small or really large values of a predictor

- This means our residuals start out small but then keep getting bigger → bad!
- predictions for small values of x are good, but predictions for large x are bad

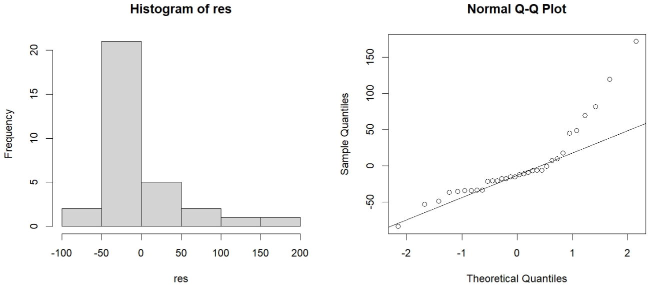




Normal QQ Plot

A Normal Q-Q plot (Quantile - Quantile) is useful for seeing if our residuals follow a Normal distribution.

- Normal QQ Plot compares the quantiles of our residuals to what we would expect of a Normal distribution that has the same variance as our residuals ($\sigma^2 = \text{MSE}$)



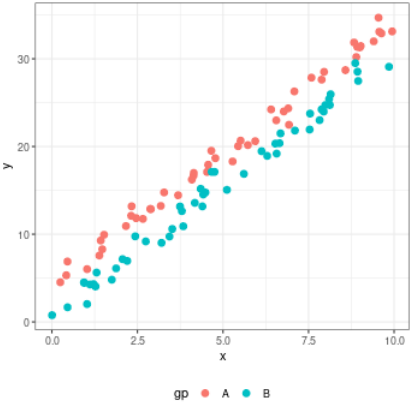
- Skewed residuals → most of the time residuals are positive/negative (bad), sometimes **really** far off in the other direction (very bad)
- straight line → Normal distribution seems OK

Part 2: Investigating Patterns

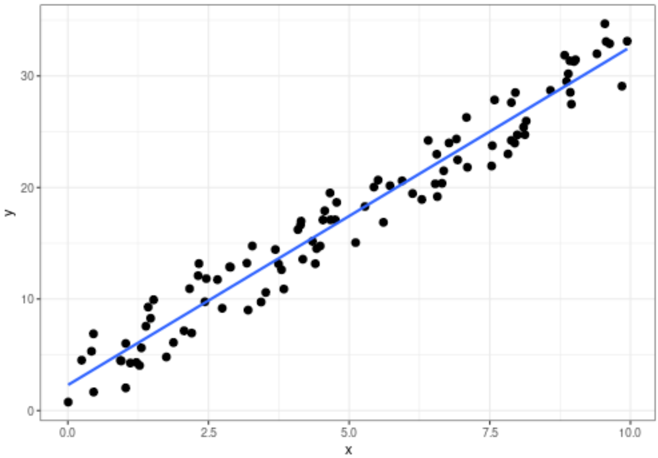
Considering new covariates

Suppose I have:

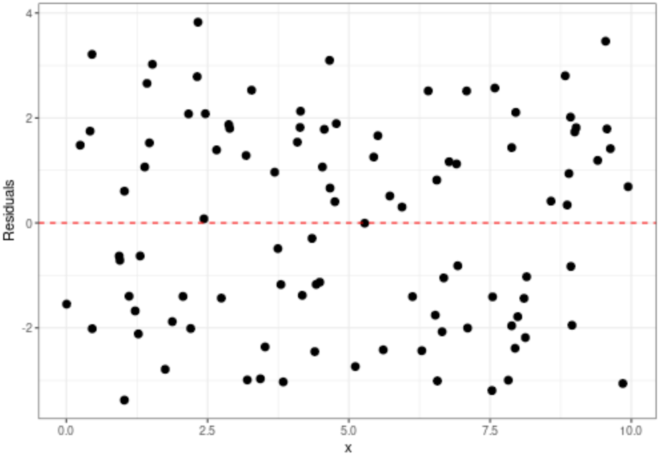
- Quantitative outcome y
- Quantitative predictor X
- Categorical predictor gp



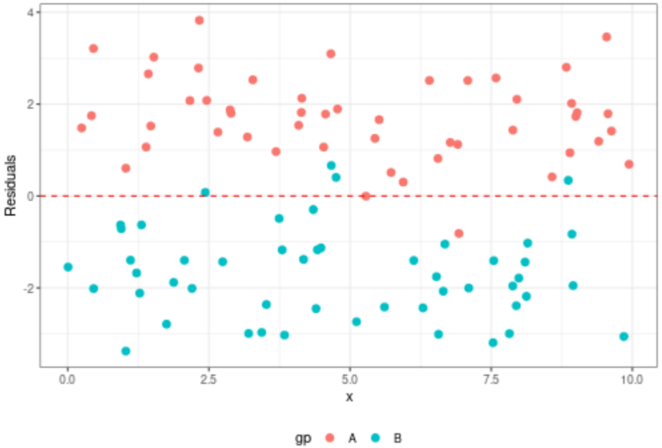
Considering new covariates



Considering new covariates

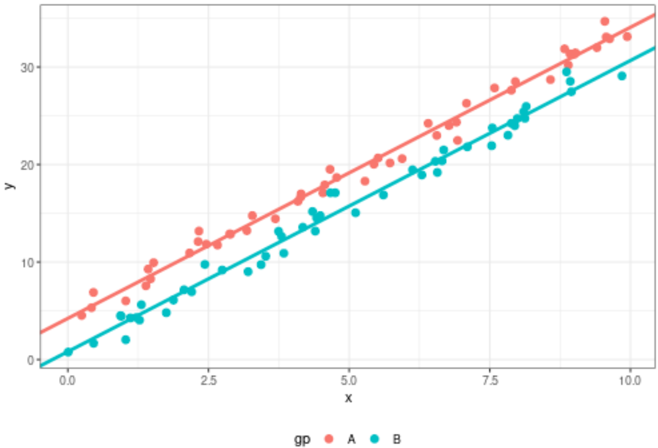


Considering new covariates



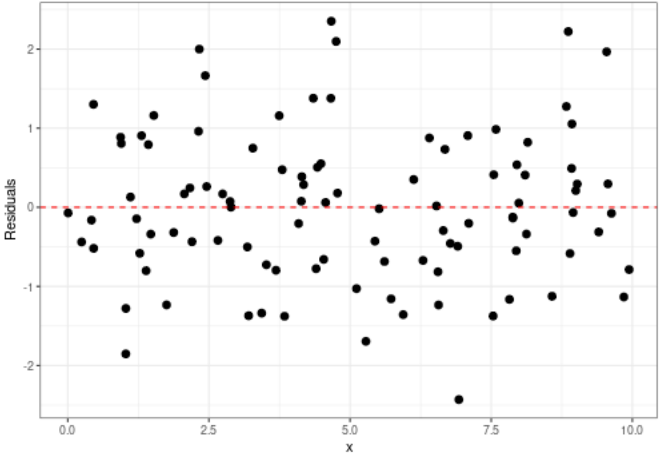
- Nearly all 'A' observations are under-predicted, all 'B' residuals over-predicted
- we could use the original scatterplot + color by gp to see pattern
  - residual plot is easier to quickly cycle through other variables to see patterns

Considering new covariates



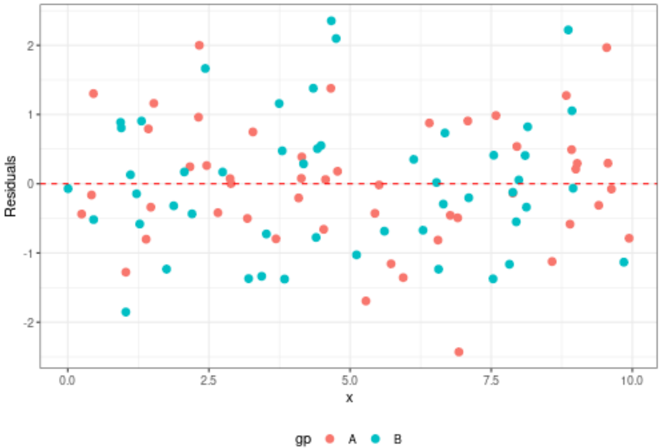
Considering new covariates

these residuals are from the model that *also* includes the gp variable



Considering new covariates

if we color by 'gp' we see that the pattern is now random about 0



- indicates this model does better than previous one

Correlated Covariates

Consider a simple linear model in which a covariate X is used to predict some value y

$$\hat{y} = \hat{\beta}_0 + X\hat{\beta}_1$$

The residuals associated with this describe the amount of variability that *is yet to be explained*

$$e = y - \hat{y}$$

The idea is to find new covariates *associated* with this residual, in effect "mopping up" the remaining uncertainty

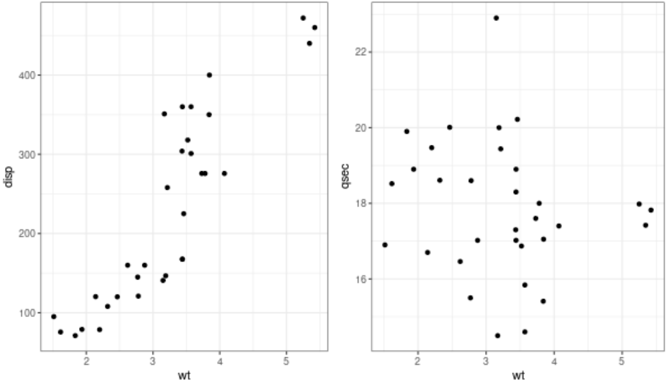
Considering new covariates

Wednesday we considered an example predicting vehicle fuel economy (mpg) with three separate models:

1. Using weight
2. Using weight and engine displacement
3. Using weight and quarter mile time (in seconds)

Correlated Covariates

Let's say I have a regression using wt to predict mpg. We are looking for a new variable to add to the model. Which of these would be better to use?



- because wt and disp are correlated, much of the info in disp is already contained within wt → probably not much improvement if we add it
- rephrased: knowing about wt already gives us a good idea of disp values → disp is not useful if we are already using wt

Correlated Covariates

Predicting mpg with wt

```
> lm(mpg ~ wt, mtcars) %>% summary()
             Estimate Std. Error t value      Pr(>|t|)
(Intercept)    37.285      1.878   19.86   < 0.000002 ***
wt             -5.344      0.559   -9.56     0.000013 ***
R-squared = 0.75
```

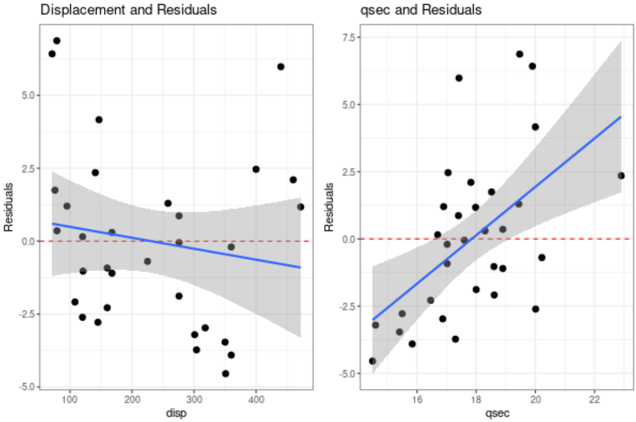
Add displacement to original

```
> lm(mpg ~ wt + disp, mtcars) %>% summary()
            Estimate Std. Error t value      Pr(>|t|)
(Intercept) 34.96055    2.16454   16.15  0.000000049 ***
wt          -3.35083    1.16413   -2.8        0.0074 **
disp        -0.01772    0.00919   -1.93       0.0636 .
R-squared = 0.78
```

Add qsec to original

```
> lm(mpg ~ wt + qsec, mtcars) %>% summary()
            Estimate Std. Error t value        Pr(>|t|)
(Intercept)   19.746      5.252    3.76         0.00077 ***
wt            -5.048      0.484  -10.43 0.000000000025 ***
qsec           0.929      0.265    3.51         0.00150 **
R-squared = 0.82
```

Residual Plots



- both of these residuals are made with model that does not use either disp or qsec
- We just saw 'qsec' would be better to add to the model $\rightarrow$ corresponds to a linear pattern in the residuals

Key Takeaways

1. Number of assumptions for linear model
   - Linearity
   - Normal errors
   - Homoscedasticity
2. Residual plots can help determine which new variables to add to model
3. Examining errors is an effective way to test assumptions