More on Standard Deviation

Interpretations, Variance, and Standardizing

Grinnell College

Fall 2025

Outline

We've talked about confidence intervals and hypothesis testing the last few weeks, which are incredibly useful tools. But there are other aspects to data analysis and other types of questions we may want to answer. We are going to return to variable associations and learn how to make *predictions*

We are going to spend a bit of time today getting a better understanding of variability

- How is it defined
- Relationship between variance and standard deviation
- What is it used for?
  - Dispersion
  - Uncertainty
  - Prediction

Previously the idea of variance helped us quantify statements such as, "this is the *best guess* we have" (CIs)

Definitions

Variance

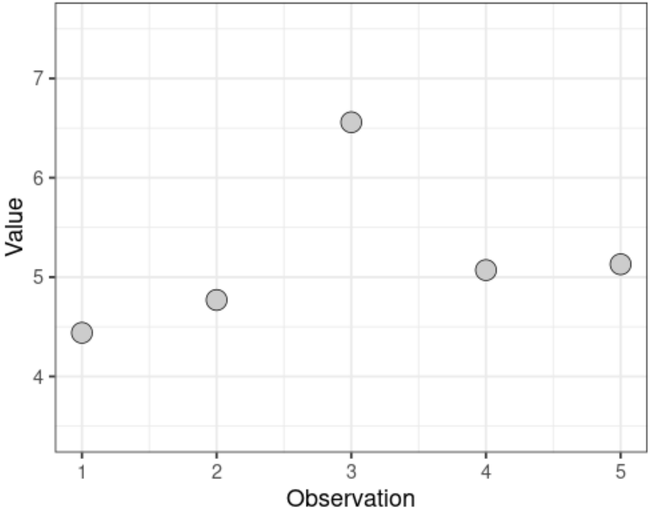
$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

Standard Deviation

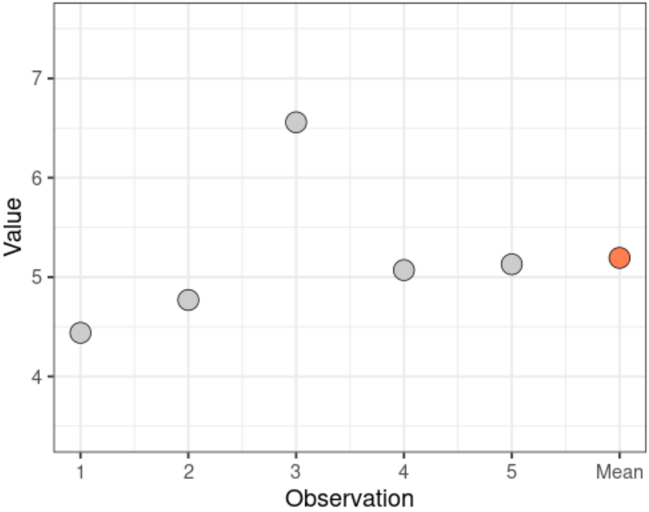
$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

The *sample* std. dev. is most often denoted with s. If we are instead referring to the std. dev. for the entire population, we will denote it as σ.

Just points

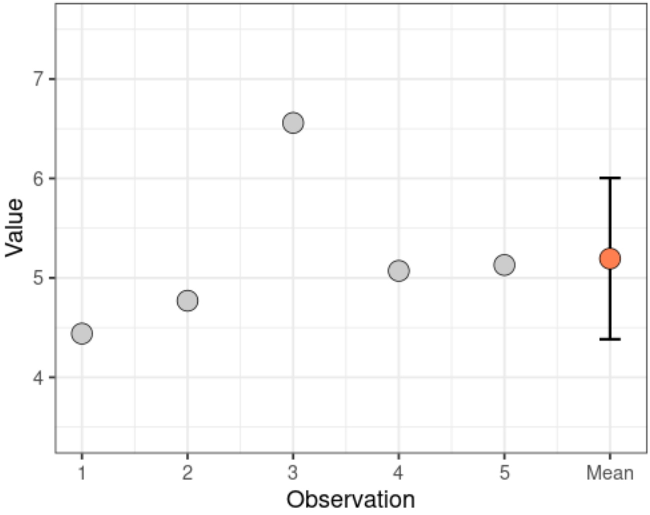


Just points



Just points

Here $n = 5$, $\overline{x} = 5.19$ and $\hat{\sigma} = s = 0.81$



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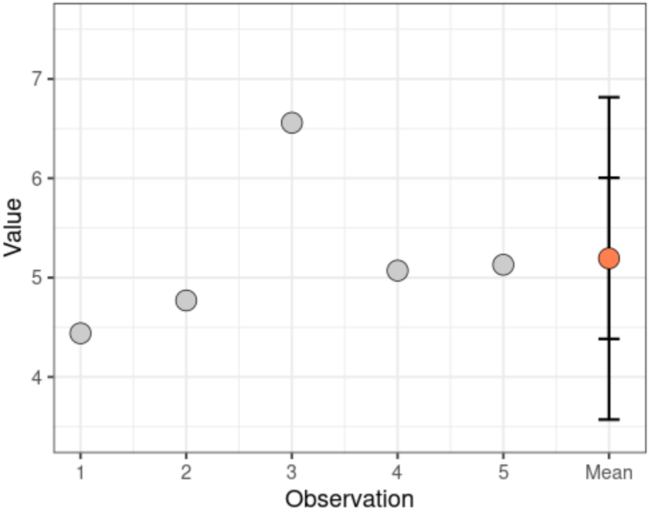
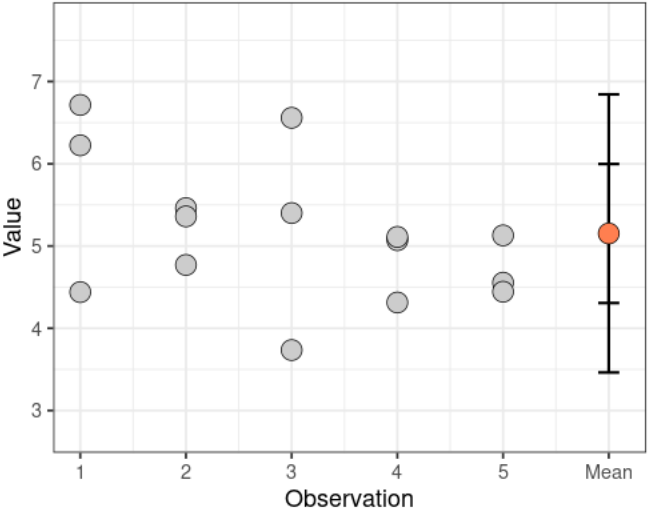


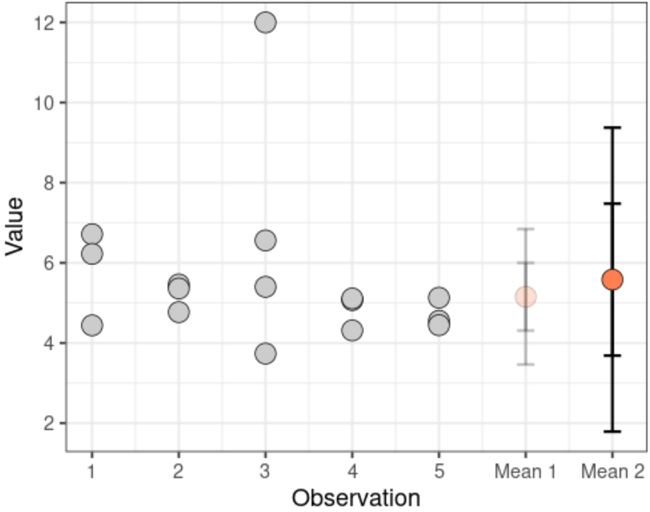
Just points

Note that the standard deviation is not necessarily affected by the number of observations. Here $n = 10$, $\overline{x} = 5.15$, $\hat{\sigma} = 0.83$ ($\approx$ same $\overline{x}$, $\hat{\sigma}$ as before)



Outlier

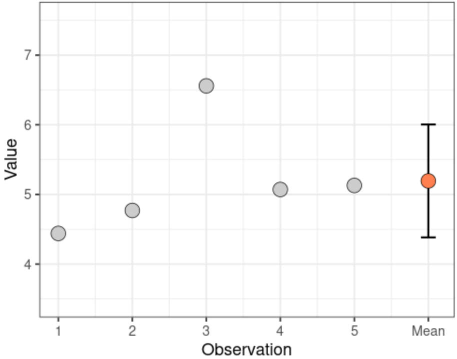
Outliers make the standard deviation larger. Same data + outlier
Now $n = 11$, $\overline{x} = 5.6$ and $\hat{\sigma} = 1.9$



Interpretation

The direct interpretation of standard deviation is "the average deviation/distance of observations to the mean"

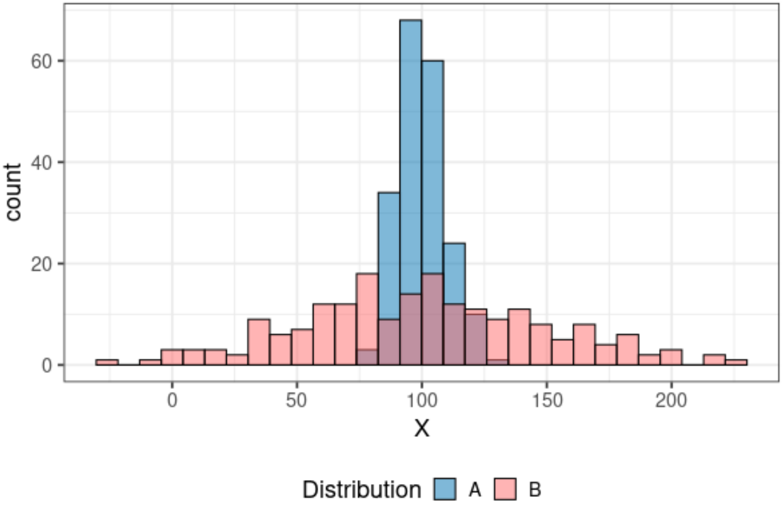
$n = 5$, $\overline{x} = 5.19$ and $\hat{\sigma} = s = 0.81$



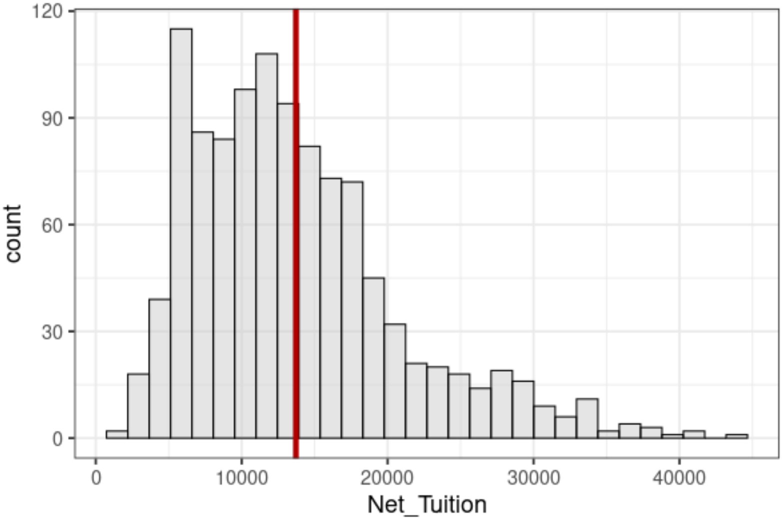
s = .81 $\rightarrow$ average deviation of observations from the mean of 5.19 is 0.81
$\rightarrow$ observations are 0.81 away from the mean, on average

Dispersion

Standard Deviation is a measure of spread. We can use it to compare distributions. Both of these have $\mu = 100$

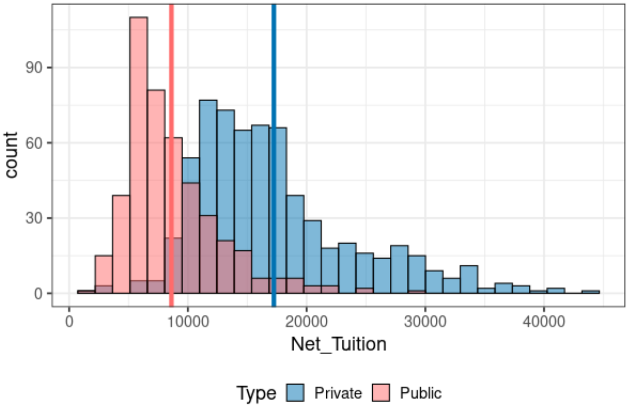


Better Centers?



$$\overline{x} = \$13713,\ \hat{\sigma} = \$7208$$

Better Centers?



$$\overline{x}_{public} = \$8615,\ \hat{\sigma}_{public} = \$3957$$
$$\overline{x}_{private} = \$17244,\ \hat{\sigma}_{private} = \$6829$$

Std. dev's. of both groups are lower than the overall std. dev. of when they were combined (splitting by type gives better predictions)

Working with Mean and Std. Dev.

Facts:

- The average height of males in the US is 69 inches with a standard deviation of 2.8 inches
- The average height of females in the US is 64 inches with a standard deviation of 2.4 inches

Who would be considered taller, relative to their sex: a male who is 72 inches tall or a female who is 66.5 inches tall?

Z-scores

A **z-score** or **standardized score** is a measurement that describes an observation's *value* relative to the mean and standard deviation of a group

$$z_i = \frac{x_i - \mu}{\sigma}$$

In particular, there are two informative attributes related to a z-score:

1. The *sign* of the z-score tells us if the observation is above or below the group mean
2. The *magnitude* of the z-scores tells us how many standard deviations away from the mean an observation is

Z-Score Comparisons

Z-score interpretation: The value we get is the number of standard deviations the value is away from the mean

- positive z-scores are larger than the mean
- negative z-scores are smaller than the mean

Examples

- If $z = 1.5$, then the observation is 1.5 standard deviations larger than the mean
- If $z = -1$, then the observation is 1 standard deviation less than the mean

Test Example (ACT vs SAT)

- The average score on the ACT English exam is 21.0 with a standard deviation of 4.0.
- The average score on the SAT Verbal exam is 520 with a standard deviation of 100.
- Kala scored a 27 on the ACT English exam.
- Nia scored a 770 on the SAT Verbal exam.

$$z_{Kala} = \frac{27 - 21}{4} = 1.5 \qquad z_{Nia} = \frac{770 - 520}{100} = 2.5$$

Who scored better? Nia, since Nia's z-score is larger

Height Comparison

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MCAT Scores

Based on data from March 2023-April 2024, the average total MCAT score was 501.03, with a standard deviation of 10.961, giving us the following summary statistics:

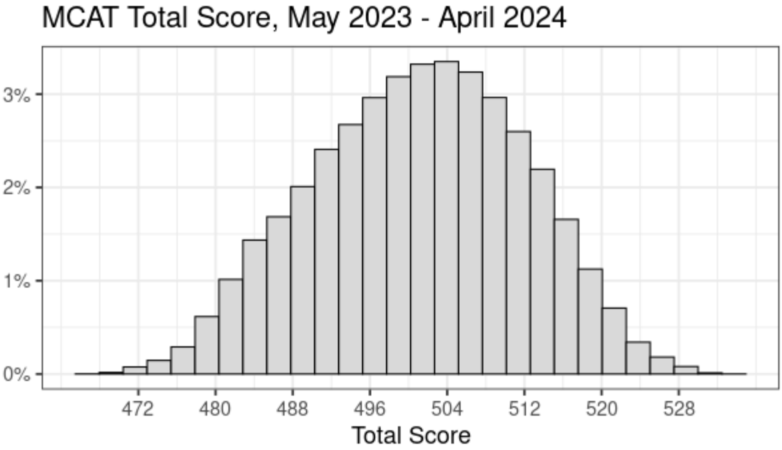
$$\mu = 501.03, \quad \sigma = 10.96$$

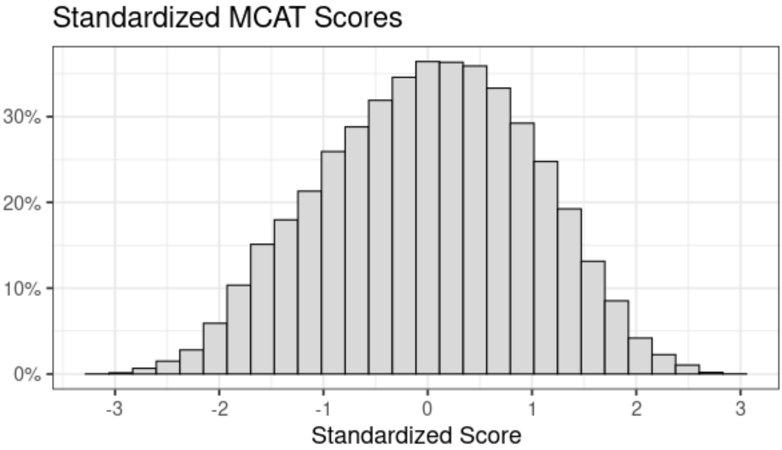
For an MCAT score of 525, we find a standardized value of

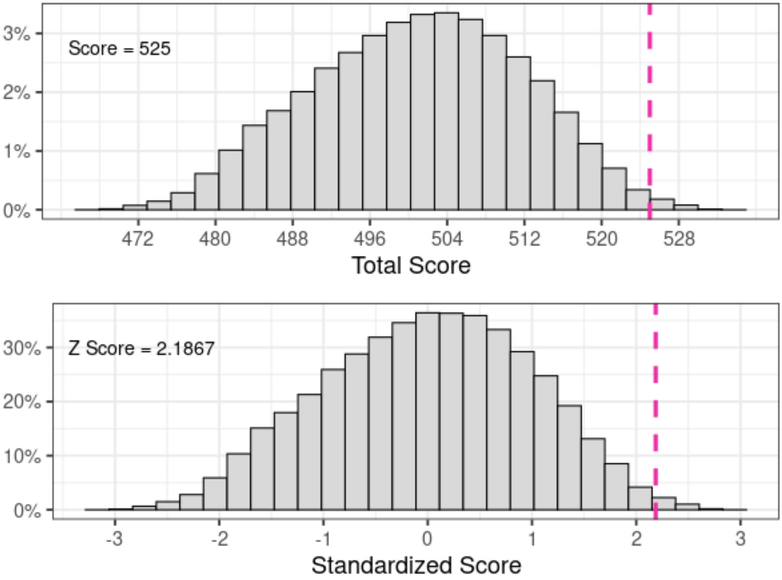
$$z = \frac{525 - 501.03}{10.96} = 2.18$$

This tells us that:

1. The observation is greater than the mean, as it is positive
2. The observation is 2.18 standard deviations greater than the mean


MCAT Total Score, May 2023 - April 2024
3%
2%
1%
0%
472
480
488
496
504
512
520
528
Total Score


Standardized MCAT Scores
30%
20%
10%
0%
-3
-2
-1
0
1
2
3
Standardized Score


Score = 525
3%
2%
1%
0%
472
480
488
496
504
512
520
528
Total Score
Z Score = 2.1867
30%
20%
10%
0%
-3
-2
-1
0
1
2
3
Standardized Score