

Confidence Intervals

What can we do with Standard Error?

Grinnell College

Normal distribution

- ▶ unimodal + symmetric bell-curve
- ▶ probabilities

Central Limit Theorem:

1. If variable X has mean μ and std.dev. σ , and
2. If the number of observations in the sample (n) is large
3. then the sampling distribution for \bar{X} (sample mean) is Normal with mean μ and standard error σ/\sqrt{n} .

$$\bar{X} \sim N(\mu, \sigma^2/n)$$

Review – Sources of Variation

Pop. Standard Deviation: Description of the variability in our *population*. It is often denoted σ

Sample Standard Deviation: Description of the variability in our *observations* (sample). It is often denoted s

Standard Error: Description of variability in our *estimates* of a parameter (such as the mean). We will denote standard error as SE , with $SE = \sigma/\sqrt{n}$, where n is the number of observations in our sample

Outline

We saw that the statistic is not going to be exactly equal to the parameter

- ▶ sampling bias
- ▶ sampling variability

So... we can't just provide a single value for our estimate of the parameter

- ▶ We also need to quantify how far away our guess is

This is why we came up with the *standard error* (SE), now we need to figure out how to use it.

Goal: We are going to spend today learning how to estimate population means

Example – COVID Vaccines

According to the U.S. Census Bureau, as of October 11, 2021:

"83.3% (+/- 0.5%) of U.S. adults 18 years and older have received at least one dose of a COVID-19 vaccine." This is based on a representative sample of civilians aged 18 and over. "Margins of error shown at 90% confidence."

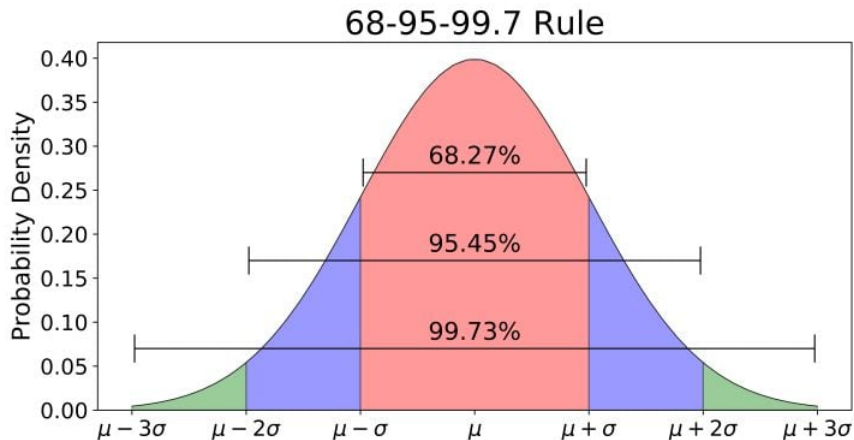
What does margin of error mean?

For the rest of these slides, our goal is to determine the mean of a *population*.

We cannot rely on only our **point estimate** \bar{X} , but perhaps we can find a range of reasonable values that looks like:

Point Estimate \pm Margin of Error

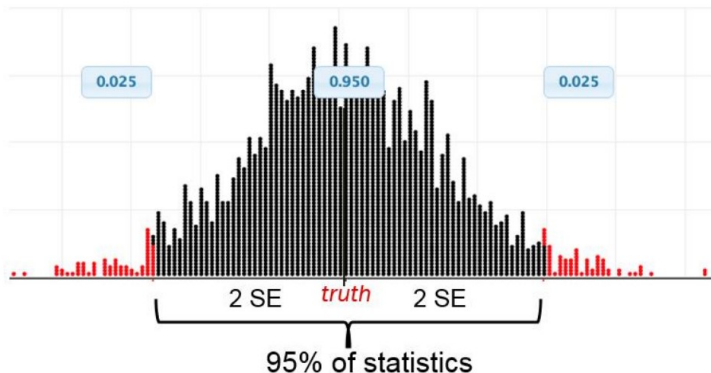
A good place to start:



Good place to start:

The sampling distribution for the sample mean looks $N(\mu, \sigma^2/n)$

- ▶ Central Limit Theorem



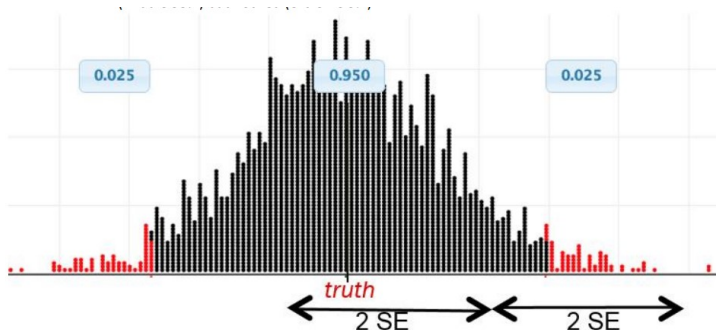
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In the sampling distribution, 95% of statistics will be within $2 \times \text{SE}$ of the pop. mean μ

Good place to start:

The sampling distribution for the sample mean looks $N(\mu, \sigma^2/n)$

- ▶ Central Limit Theorem



Equivalently: The interval 'statistic $\pm 2 \times \text{SE}$ ' will contain μ for 95% of the statistics

Confidence Interval

We are going to call this interval a "95% Confidence Interval":

- ▶ 95% comes from the fact that 95% of statistics are within 2SE's of the mean
- ▶ Confidence refers to the fact that this is a range of plausible values for the parameter

Formula for a 95% confidence interval for estimating a pop. mean (μ) is:

$$\bar{x} \pm 2 \times SE$$

Confidence Interval

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$$\bar{x} \pm 2 \times SE$$

Margin of Error tells us how wide our interval is.

- ▶ ME = half the width (or length) of the interval
- ▶ for 95% CI \rightarrow ME = $2 \times SE = 2 \frac{\sigma}{\sqrt{n}}$

This makes our (final) formula for a 95% confidence interval for estimating a pop. mean (μ):

$$\bar{x} \pm 2 \times \frac{\sigma}{\sqrt{n}}$$

CI Interpretation

Remember our goal: we are trying to estimate the *population mean*

Confidence Interval Interpretation

- ▶ mention confidence level
- ▶ specify the values we got
- ▶ use context for the population mean when able

"We are 95% confident that (the population mean) is between (lower value) and (upper value)."

Example – Movie Budgets

Hollywood movie budget data: $\mu = 51.38$, $\sigma = 57.93$

From a sample of 50 movies we find $\bar{x} = 45.65$.

Construct a 95% Confidence Interval for the pop. mean movie budget.

Example – Movie Budgets

Hollywood movie budget data: $\mu = 51.38$, $\sigma = 57.93$

From a sample of 50 movies we find $\bar{x} = 45.65$.

Construct a 95% Confidence Interval for the pop. mean movie budget.

- ▶ When we know σ we can use the formula directly

$$\bar{x} \pm 2 \frac{\sigma}{\sqrt{n}} \rightarrow 45.65 \pm 2 \times \frac{57.93}{\sqrt{50}} \rightarrow 45.65 \pm 16.39$$
$$(29.26, 62.04)$$

Example – Movie Budgets

The 95% CI for the population mean is (29.26, 62.04).

Interpretation:

We are 95% confident that (the population mean) is between (lower value) and (upper value).

Example – Movie Budgets

The 95% CI for the population mean is (29.26, 62.04).

Interpretation:

We are 95% confident that the population mean movie budget is between 29.26 and 62.04 million dollars.

Example – Fish Mercury Levels

Data was collected from 53 lakes in Florida. For each lake, the mercury level (parts per million) was computed for a large mouth bass.

Based on this sample of fish, the mean mercury level was 0.527 ppm with a standard deviation of .118

Construct a 95% CI for pop. mean:

- ▶ Issue: we don't know σ

Example – Fish Mercury Levels

Data was collected from 53 lakes in Florida. For each lake, the mercury level (parts per million) was computed for a large mouth bass.

Based on this sample of fish, the mean mercury level was 0.527 ppm with a standard deviation of 0.118

Construct a 95% CI for pop. mean:

$$\bar{x} \pm 2 \frac{\sigma}{\sqrt{n}} \rightarrow \bar{x} \pm 2 \times \frac{s}{\sqrt{n}} \rightarrow 0.527 \pm 2 \times \frac{0.118}{\sqrt{53}}$$
$$(0.495, 0.560)$$

We are 95% confident that the true pop. mean mercury level of fish in Florida lakes is between 0.495ppm and 0.560ppm

More on "Confidence"

We are 95% confident that...

Let's really dig into what this 'confidence' part means

More on "Confidence"

A **confidence interval** is an interval that has the following properties:

- ▶ It is constructed according to a procedure or set of rules
- ▶ It is made with the intention of giving a plausible range of values for a *parameter* based on a *statistic*
- ▶ There is no probability associated with a confidence interval; *it is either correct or it is incorrect*

More on "Confidence"

Consider the confidence interval that we constructed in the movie budget example.

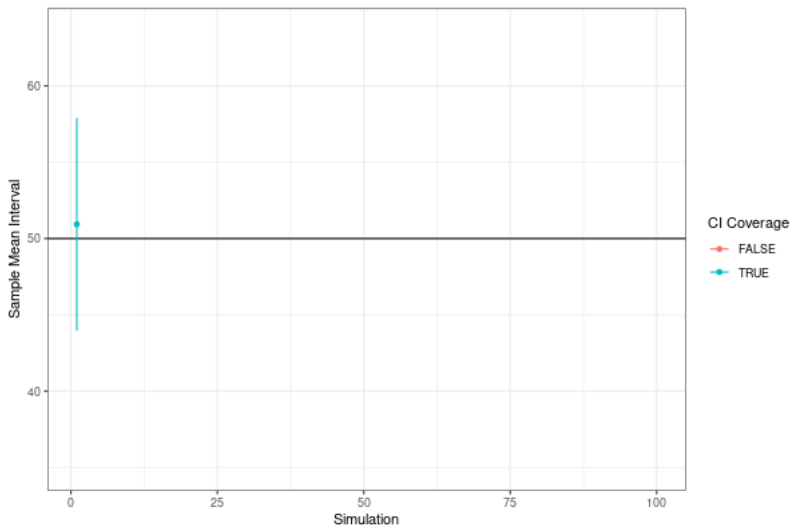
- ▶ It was constructed according to the procedure
Point estimate \pm Margin of Error
- ▶ It was made to present a reasonable range of values for the *parameter* μ as estimated by the *statistic* \bar{X}
- ▶ The interval was (29.26, 62.04). As our true mean is $\mu = 51.38$, this interval *is* correct in the sense that it *contains* our true parameter

More on "Confidence"

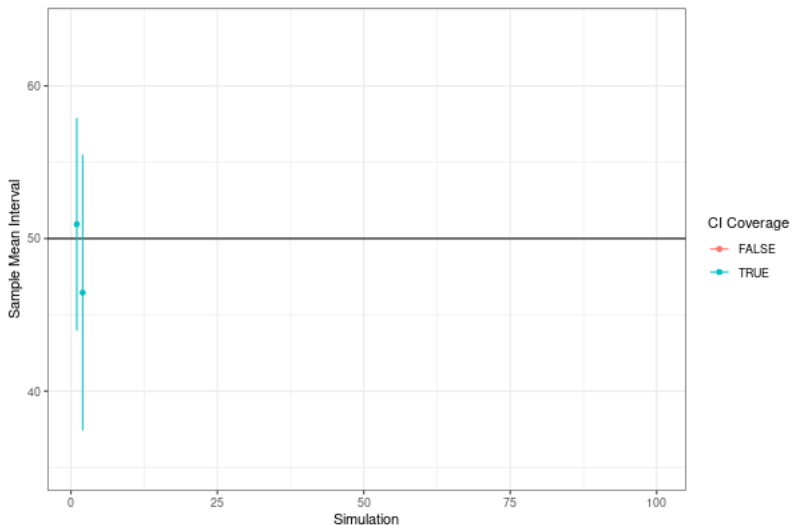
When we say something has a 95% confidence interval, what we mean is:

The process that constructed this interval has the property that, on average, it contains the true value of the parameter 95 times out of 100

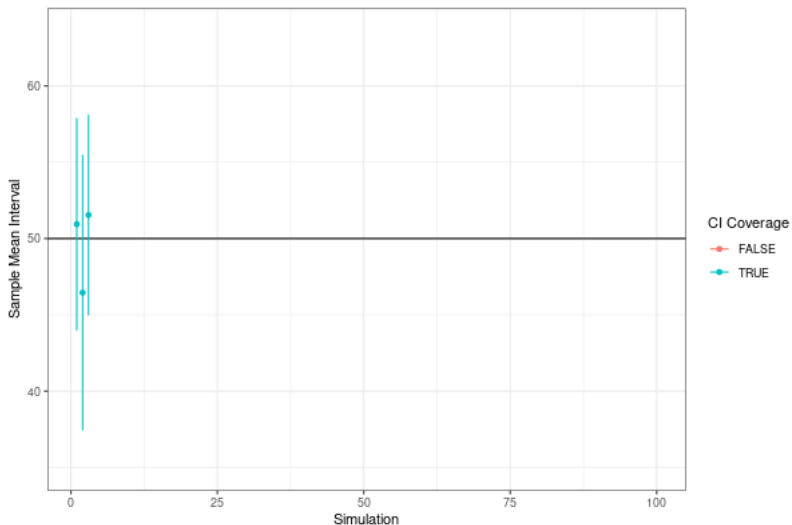
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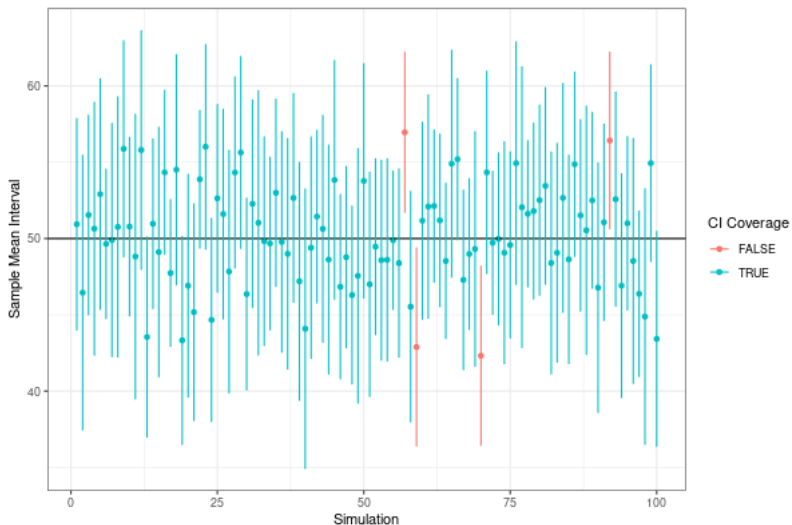
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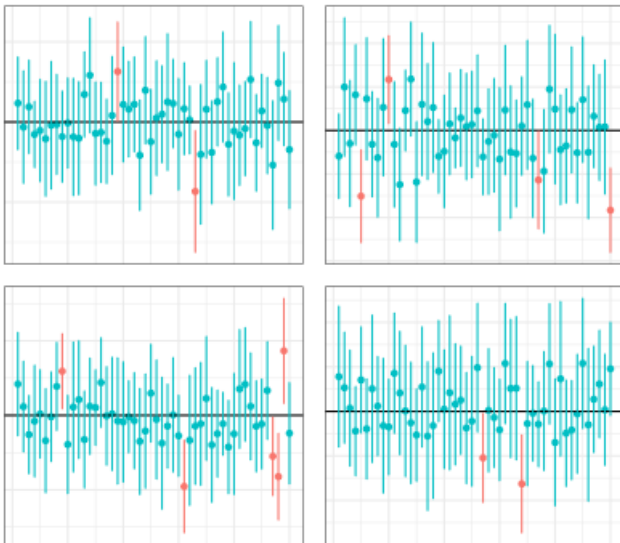
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Coverage



Coverage



Confidence Intervals

To be absolutely clear: we will **never** know if the confidence interval we construct contains the true value of the parameter

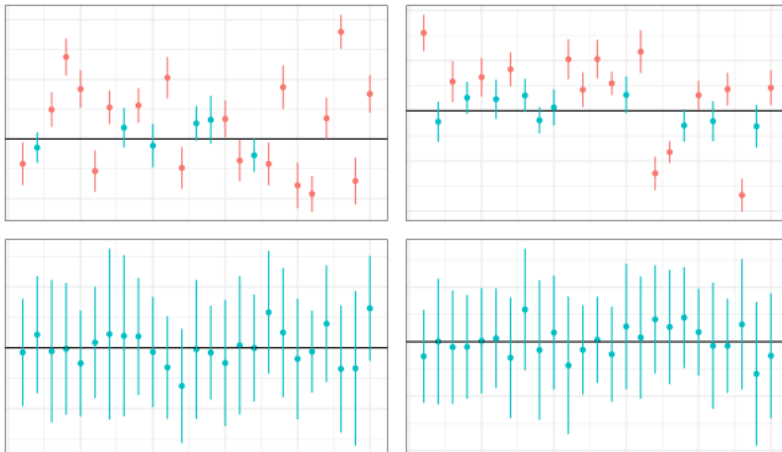
This is kind of like throwing a dart but never seeing the target

This is the nature of statistical inference

- ▶ we can describe properties of the *process* that created our intervals
- ▶ we can never conclusively speak about the interval itself

Confidence Intervals

It is also worth observing that we can *alter* our process to achieve different results. There is a tradeoff between how frequently we are correct and how much uncertainty we allow in our prediction



Common Misinterpretations

These are some common ways people interpret CI's that are absolutely **not** correct:

- ▶ A 95% confidence interval contains 95% of the data in the population.
- ▶ “I am 95% sure that the mean of the sample will fall within a 95% confidence interval for the mean.”
- ▶ “95% of all sample means will fall within this 95% confidence interval.”
- ▶ “The probability that the population parameter is in this particular 95% confidence interval is 0.95.”

Generalizations

Confidence Intervals quantify sampling variability

- ▶ Range of plausible values for the statistic
- ▶ Range is determined by how much the statistics vary from sample to sample

Confidence Intervals **DO NOT** account for bias in the samples

- ▶ We will never be able to quantify the bias in our samples
- ▶ It is important to mention possible sources of bias in our final conclusions