

Hypothesis Testing pt. 2

More on Null Distributions and Test-statistics

Grinnell College

Hypothesis Testing: formal technique for answering a question with two competing possibilities

Null Hypothesis: represents a skeptical view or a perspective of no difference

- ▶ H_0 : 'parameter' = (some value)

Alternate Hypothesis: what the researchers actually want to show with the study

- ▶ H_A : 'parameter' [$<$ / $>$ / \neq] (some value)
- ▶ choose the sign to match the research question

This is the general outline we will follow for Hypothesis Testing

1. Define hypotheses
2. Simulate what the parameter looks like under H_0
3. See how our statistic compares to this
4. Compute a p-value
 - ▶ do the results look unlikely if H_0 is true?
5. Interpretations / Conclusions

Note: Points 2 and 3 can be combined with the creation of a 'test-statistic'

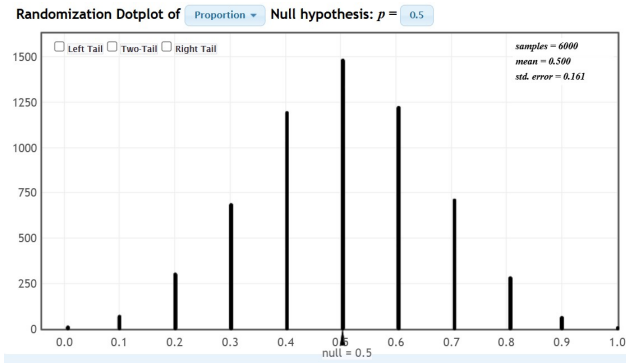
Null Distribution

Null Distribution

The distribution of the statistics if the null hypothesis is true

- ▶ simulates what the null hypothesis looks like
- ▶ use this to compute p-values

We looked at the coin-flip scenario.



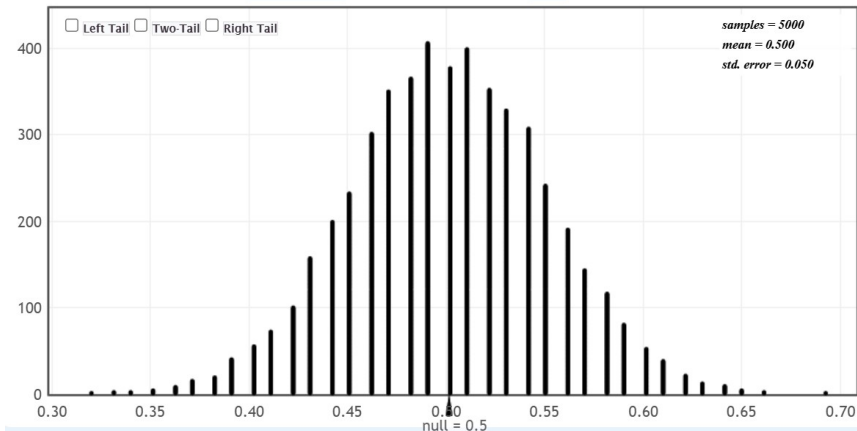
Null Distribution

What if we flipped 100 coins instead of 10

- ▶ Would getting $\hat{p} = .8$ be more common or less common?

Null Distribution

This represents ' $H_0: p = 0.5$ ', when the sample size (# of flips) is 100



- ▶ What do we see?
- ▶ Are large or small values of \hat{p} more/less common?

Null Distribution

The **null distribution** will look very similar to the sampling distribution stuff we saw before

- ▶ this time it is simulating the null hypothesis
- ▶ is **not** tell us what the population parameter looks like
- ▶ for means and proportions this looks like a Normal curve

These distributions looks Normal when certain conditions are met. Very similar to what we had when we were using confidence intervals.

Null Distribution – Proportion

Conditions for a proportion hypothesis test:

- ▶ Random Sample
 - ▶ this doesn't affect our null distr. but makes sure answers are accurate
- ▶ $np_0 > 10$
- ▶ $n(1 - p_0) > 10$

With the conditions met, the null distribution for \hat{p} looks like this:

$$\hat{p} \sim N(p_0, \sqrt{\frac{p_0(1 - p_0)}{n}})$$

Note: we have p_0 's in the distribution because we are simulating what the null hypothesis looks like

Test Statistics

$$\hat{p} \sim N(p_0, \sqrt{\frac{p_0(1-p_0)}{n}})$$

The test-statistic saves us from having to plot this Normal distribution every time. We will *standardize* \hat{p} to make the distribution simpler.

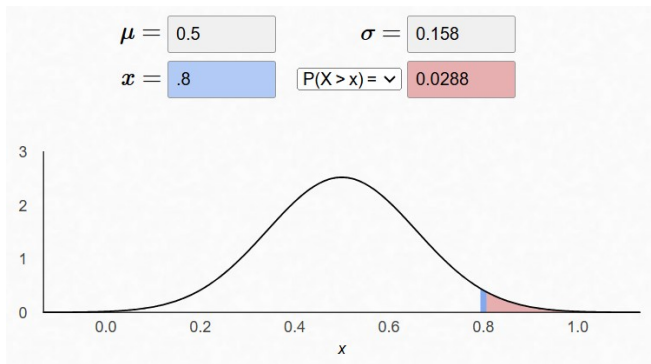
$$\underbrace{\frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}}_{\text{Test Statistic}} \sim N(0, 1)$$

- ▶ the whole term on the left is the Test-statistic
- ▶ we can compute this value and we know it follows $N(0,1)$

Coin Flip Example

For our scenario of 10 flips...

- ▶ $\hat{p} = 8/10 = 0.8$
- ▶ $p_0 = 0.5$
- ▶ $\sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{10}} = 0.158$

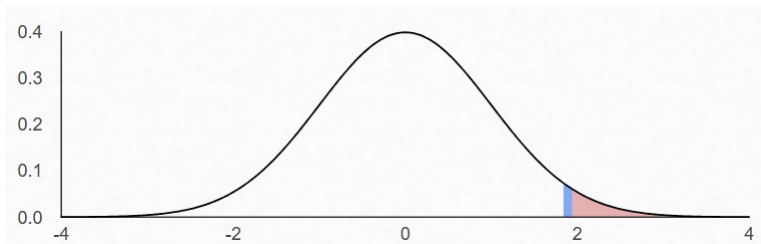


- ▶ can use Null distr. and \hat{p} to get p-value

Coin Flip Example

For our scenario of 10 flips...

- ▶ $\hat{p} = 8/10 = 0.8$, $p_0 = 0.5$
- ▶ $Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.8 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{10}}} = 1.897$



- ▶ can use $N(0,1)$ and Test-statistic Z to get p-value

From here on out, we will place almost all of our emphasis on Test-statistics instead of elaborately detailing what the null distribution should look like.